An Affine Parameter Dependent Controller for an Autonomous Helicopter at Different Flight Conditions

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Abstract: In this paper we address the design of a controller that achieves stabilization and reference tracking at different flight conditions for an unmanned helicopter. The controller proposed is in the form of an H-infinity gain-scheduler, and is used for stabilization and reference tracking, for the 4 axis autopilot. (heave, pitch, roll and yaw control) A nonlinear helicopter model has been built, trimmed and linearized at different flight conditions. Based on the linearized models an approximate affine parameter dependent model has been constructed. Then, a linear parameter dependent controller is synthesized which stabilizes the affine parameter dependent helicopter model. By doing so, a single controller achieves stabilization and reference tracking of a family of linear models by scheduling the controller gains based on the online measurement of the scheduling parameter, which is the forward velocity. Moreover, the affine parameter dependent controller is fitted into the nonlinear helicopter model. It is seen that this single parameter dependent controller successfully stabilizes the nonlinear helicopter model at different flight conditions.

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1. INTRODUCTION

Aerial vehicles have complex nonlinear behavior which changes significantly at different flight conditions. Generally speaking, control design is based on classical design rules and nonlinear variations on the system dynamics are usually handled by the standard gain scheduling techniques by simply tuning the controllers at different flight conditions. Specifically, helicopters are highly unstable and complex systems whose dynamics changes with variations on the flight conditions. Thus the need for a gain scheduled controller becomes more evident.

In (Bates D. and Postlthwaite I., 2002), a mixed sensitivity $H_\infty$ controller is designed based on a linearized model of a Bell 205 helicopter as a design example. In (Postlthwaite I., Prempain E., Turkoglu E., Turner M. C., Ellis K., and Gubbels A.W., 2005), $H_\infty$ controllers are designed using the linearizations extracted from nonlinear model of the Bell 205 helicopter. It has been noted that $H_\infty$ controllers provided satisfactory stabilization of the helicopter and yielded desirable handling qualities in flight testing. Moreover, in (Luo C., Liu R., Yang C., and Chang Y., 2003), an $H_\infty$ flight control system is also designed to improve its stability, maneuverability and agility. Resulting $H_\infty$ flight control system is then fitted into the nonlinear model of a helicopter to simulate nonlinear dynamic response.

In (Pei H., Hu Y., and Wu Y., 2007), a robust gain-scheduling algorithm is introduced by employing local multivariable LQR controllers which are designed at each operation point, for a given performance index. For the controllers between two neighbor operating points, a scheduling scheme is employed for the control gain interpolation. In (Abulhamitibilal E. and Jafarov M., 2011), a flight control system with gain scheduled LQ optimal controllers are designed based on look-up tables. Gain scheduling is simply accomplished by changing the values of the control matrix according to predefined conditions. However, it should be stressed that the creation of separate control laws is laborious compared to the self scheduling method which forms an automatic interpolation law.

In the 90’s Linear Parameter Varying (LPV) control has been presented as a reliable alternative to classical gain-scheduling for multivariable systems; typical well known examples can be found in (Shamma J. S. and Cloutier J. R., 1993) and (Apkarian P., Gahinet P., and Biannic J.M., 1994). Gain scheduling is a standard method to design controllers for dynamical systems over a wide performance envelope. In (Asif A. and Smith R., 2000), LPV control design techniques are also applied to the attitude control of the X-33, a single-stage-to-orbit (SSTO) prototype vehicle. A multivariable LPV controller was designed using $H_\infty$ synthesis for F/A-18 in (Balas G.J., Mueller J.B., and Barker J., 1999). This type of controller was chosen because the change in dynamics of the aircraft state-space matrix and input matrix are approximately affine functions of forward velocity. The results show that the LPV controller performed the specified objectives and is therefore a sufficient controller for the F/A-18 model.

In (Balas, 2002) two different approaches are proposed that can be used to obtain reliable LPV models. Unfortunately,
there is no proposed way of obtaining quasi LPV models either by state transformation or function substitution for a nonlinear helicopter model. Generally, control designers use a family of linear, time-invariant (LTI) plants at different points of interest throughout the operational envelope in order to obtain a reliable LPV model. Jacobian linearization method is applicable to the widest class of nonlinear systems, since it is valid for any nonlinear system that can be linearized around its equilibrium points. Thus, Jacobian linearization approach can be used to obtain an affine parameter dependent model of nonlinear helicopter model. Based on the locally linearized family of systems, it is straightforward to construct a parameter dependent model that captures the nonlinear behavior. Hence, gain scheduling enables synthesis of global controllers based on interpolation of a family of locally linearized controllers.

In this paper, we develop a mathematical simulation model of an autonomous helicopter based on information of a Yamaha R-50 model helicopter given in (Munzinger, 1998). In Section 2, development of a minimum complexity helicopter simulation math model has been carried out in SIMULINK environment. Jacobian linearization is used to create a family of plants linearized with respect to a set of equilibrium points that represent the flight envelope of interest. The resulting model will be a local approximation to the dynamics of the nonlinear plant around a given set of equilibrium points. An affine parameter dependent system is obtained in Section 3 by using the family of linearized plants. Section 4, the mixed sensitivity $H_\infty$ controller synthesis approach is summarized. Based on the affine parameter dependent helicopter model an affine parameter dependent controller is designed in Section 5 which stabilizes the closed-loop system and provides reference tracking to the pilot reference commands in different flight conditions by scheduling the controller based on the online measurement of scheduling parameter. Finally in Section 6, the parameter varying controller is tested on all trimmed models at different equilibrium/linearization points. Lastly, the affine parameter dependent controller is fitted into the nonlinear helicopter model and it is seen that it stabilizes the system and provides reference tracking to the pilot commands.

2. NONLINEAR HELICOPTER MODEL

The helicopter is a highly unstable system for which a controller is needed to be able to achieve flight. The controller should make the helicopter model stable within the entire flight envelope in a simulation. As the main objective is to control the dynamical behavior of the helicopter, it is necessary to derive a representative model that reacts in the same manner as a real helicopter. In that respect a very accurate model is preferred, but the complexity increases with model accuracy. A highly complex model will limit the capability of real-time simulation. The nonlinear model must approximate the behavior of the actual helicopter system as closely as possible. The modeling approach mainly based on the NASA report in (Heffley R.K. and McNich M.A., 1988).

The helicopter is considered to be a rigid body, free to move in three directions and to rotate about all three axes, hence having 6 degrees of freedom (DOF). Basically, two different helicopter reference-frames are defined throughout the helicopter modeling: body fixed reference frame and earth fixed frame. All of these are right-handed coordinate systems. For deriving equations of motion, it is convenient to define a body fixed frame following the attitude and position of the helicopter. The $x$ axis of the body frame is defined to point in the helicopter longitudinal direction. The $y$ axis is defined to point to the right (lateral direction) when seen from above, and the $z$ axis downwards and perpendicular to the other axes. The helicopter uses the main and tail rotor to perform these movements. By altering the pitch of the blades, the magnitude and orientation of the resulting thrust force can be controlled. The position and attitude of the helicopter are controlled through the following 4 control inputs commanded by the pilot: collective, longitudinal cyclic, lateral cyclic and pedal.

The modeling of the helicopter will be performed using a top-down principle. The entire model consists of three parts which comprise the nonlinear helicopter model. The nonlinear model is implemented in SIMULINK, for testing of the devised controller.

2.1 Thrust and Flapping equation

The blades of the main rotor generate the needed lift to the helicopter. This is done by accelerating the air downwards and thereby generates a counter force upwards. Main rotor thrust can be given as

$$ T_{MR} = (\omega_b - v_i)\frac{\rho \Omega R^2 a B c}{4} \quad (1) $$

and induced flow $v_i$ depends on main rotor thrust as follows

$$ v_i^2 = \sqrt{\frac{(\hat{\beta}^2)^2}{2} + \frac{(T_{MR})^2}{2 \rho A} - \hat{\beta}^2} \quad (2) $$

where

$$ \omega_b = \omega_r + \frac{2}{3} \Omega R u_{coll} $$
$$ \omega_r = w + (\beta c + \mu)u - \beta c v $$
$$ \hat{\beta}^2 = u^2 + v^2 + (\omega_r - v_i). $$

A more detailed analysis on main rotor thrust equations can be found in (Munzinger, 1998) and (Hald U.B., Hesselbaek M.V., Holmgaard J.T., Jensen C.S., Jakobsen S.L., and Siegumfeldt M., 2005). The main rotor thrust equations need to be solved iteratively, since $T_{MR}$ depends on $v_i$ and vice-versa. $T_{MR}$ is calculated numerically by iterating the solutions of $T_{MR}$ and $v_i$. Iterative solution for main rotor thrust and inflow needs to be solved by iteration in each time sample. This iteration is repeated until the values of $T_{MR}$ and $v_i$ have settled. Approximately 5 iterations are enough to ensure that the values have settled as advised in (Pettersen R., Mustafic E., and Fogh M., 2005). These iterations are carried out in a single sample step time when the model is simulated in SIMULINK.

Tail rotor thrust is not modeled and it is assumed that the tail rotor thrust force can be instantaneously applied by the pedal input in order to counteract the torque made by the main rotor.
The equations for the main-rotor flapping are obtained from (Hald U.B. ,Hesselbæk M.V. ,Holmgaard J.T. , Jensen C. S. ,Jakobsen S. L., and Siegumfeldt M., 2005). The equations are valid for a teetering rotor with hinge offset.

2.2 Generation of Forces and moments

The forces acting on the helicopter create both translatory and rotary movement because the forces on the act main and the tail rotors. The forces and the moments are translated into the center of gravity.

2.3 Rigid Body Motion

The helicopter is considered as a rigid body, the forces and torques produced by the main and tail rotor are used to determine the motion of the helicopter. Rigid body equations describe the position and the translatory movement of the center of gravity relative to the earth frame. The Euler angles are used to describe the angles between the earth frame and body frame. Rigid body motion consists of 3 force and 3 moment equations in the body frame. Moreover, 3 additional equations are used to describe kinematic relationship between body frame and earth frame (Padfield, 2007).

3. PARAMETER DEPENDENT MODEL

It is well known that the helicopter dynamics greatly vary as the forward velocity changes (Padfield, 2007). The flight envelope can also be considered as the parameter space and an LPV controller could be designed as in (Balas, 2002). The parameter space is selected as forward velocity, because the change in the helicopter dynamics system matrix and input matrix are approximately affine functions of forward velocity. Although the parameter variations are not purely linear, we assumed that the parameter dependency is close to linear. This assumption enables the construction of an affine parameter dependent model from a family of linear models. By numerical inspection of the system matrix and input matrix, it is verified that the variation is close to linear. However, there are still some parameters whose variation is highly nonlinear and it is accepted that some of the information is lost in the linear parameter varying model.

A trim condition is defined as an operating point of the nonlinear model, which can be thought of as the specific flight condition for the helicopter in (Hald U.B. ,Hesselbæk M.V. ,Holmgaard J.T. , Jensen C. S. ,Jakobsen S. L., and Siegumfeldt M., 2005). In this context, a trim condition is equilibrium point of the nonlinear helicopter model, meaning that the state derivatives and input derivatives all equal zero. Therefore, trim is simply determining stable or unstable equilibrium points. Given the equilibrium points, non-linear helicopter model is linearized via Jacobian linearization based on small perturbation approach (Balas, 2002). The linearizations are carried out at a number of trim values representative of the flight envelope of interest. In our case, nonlinear helicopter model is linearized at 3 different forward speed (0,10,20 m/s forward speed) in order to find a family of linear models which capture the nonlinear helicopter model at different flight conditions. Linearized model has 8 states, 4 inputs and 4 outputs. Singular values of the open-loop plant can be seen in Fig. 1.

Fig. 1 Singular values of the open-loop plant

A family of linear models can now be expressed as a parameter dependent model as

\[
\dot{x} \approx A(\rho) x + Bu
\]

\[
y = C x
\]

where \( A(\rho) = A_0 + A_1 \rho \), \( B_0 = B_0 + B_1 \rho \) and \( \rho \) is the forward velocity. The parameter dependent model is augmented by adding first order actuator dynamics where the actuator system matrices can be given as \( A_{act} = -20 \), \( B_{act} = 1 \), and \( C_{act} = 1 \). Hence the parameter dependent model with actuator dynamics can be given as

\[
\dot{x} = A(\rho) x + Bu
\]

\[
y = C x
\]

where \( A(\rho) = \begin{bmatrix} A_0 & B_0 C_{act} \\ 0 & A_{act} \end{bmatrix} \), \( B = \begin{bmatrix} 0 \\ B_{act} \end{bmatrix} \), \( C = \begin{bmatrix} C_p \\ 0 \end{bmatrix} \) and

\[
x = \begin{bmatrix} x_p \\ x_{act} \end{bmatrix}
\]

Notice that system matrix can be transformed into an affine parameter dependent system matrix \( A(\rho) = \begin{bmatrix} A_0 + A_1 \rho & (B_0 + B_1 \rho) C_{act} \\ 0 & A_{act} \end{bmatrix} \) and input matrix \( B \) is not parameterized by \( \rho \). Moreover, the output is also derived from states so \( C \) is also independent from parameter \( \rho \). Indeed, adding actuator dynamics transfers the parameter dependency on the input matrix to the system matrix.

4. MIXED SENSITIVITY \( \mathcal{H}_\infty \) CONTROLLER SYNTHESIS

In this section, we will describe mixed sensitivity \( \mathcal{H}_\infty \) controller design. We consider the general feedback configuration shown in Fig. 1.

Fig. 2. General \( \mathcal{H}_\infty \) Control Configuration.
The closed-loop transfer matrix from $d$ to $z$ is given by a lower linear fractional transformation as $z = F_1(P_{aug}, K)d$. The standard $H_{\infty}$ optimal control problem is then equivalent to the minimization of the $H_{\infty}$ norm $F_1(P_{aug}, K)$ over all stabilizing controllers. In other words $H_{\infty}$ control design is simply based on designing a stabilizing controller $K$ such that the transfer function from $d$ to $z$ is minimized for all frequencies. Until now, we described the $H_{\infty}$ control problem in the most general setting. More specifically, we are interested in the formulation of tracking problem as a mixed sensitivity problem, thus converting the problem as an $H_{\infty}$ optimization problem. We consider the problem of synthesizing a controller $K$ to satisfy attenuation of low frequency disturbances and minimization of high frequency actuator activity. In this formulation, the exogenous input signal $d$ is reference command $r$, and the controlled $z$ variables are chosen to minimize low frequency error and high frequency control action. Details can be seen in Fig. 2.

Fig. 3. Mixed Sensitivity Optimization (tracking)

Two weighting filters now need to be introduced to reflect specifications on our control design requirements such as reference tracking of pilot commands and limiting control signals to force down the closed-loop bandwidth thus enabling a more robust control design. Two weighting filters are used in order to cast mixed sensitivity reference tracking problem in the $H_{\infty}$ framework based on the thumb rules given in (Bates D. and Postlethwaite I., 2002). $W_e$ is the error filter and the selection of the error filter based on:

- Large magnitude at low frequency: to suppress low frequency error. By doing so, steady state error is eliminated in the tracking problem.
- Low magnitude at high frequency: to allow high frequency error.

$W_d$ is the control output filter and the selection of the control output filter based on:

- Small magnitude at low frequency: to allow low frequency control outputs for stabilization and reference tracking.
- Large magnitude at high frequency: to suppress high frequency control outputs, thus force down the closed-loop bandwidth.

5. GAIN SCHEDULED CONTROLLER

In this section, we will synthesize a self-scheduling controller by using online measurements of the scheduling parameter $\rho$. Given the affine parameter dependent system $P$

$$\dot{x} = A(\rho)x + B_d(\rho)d + B_u u$$
$$z = C_x(\rho)x + D_{xd}d + D_{zu}u$$
$$y = C_y x + D_{yd}d + D_{yu}u$$

controlled by an affine parameter dependent controller $K$

$$x_c = A_c(\rho)x_c + B_c(\rho)y$$
$$u = C_c(\rho)x_c + D_c(\rho)y$$

which is scheduled by the online measurement of parameter $\rho$ where $\rho$ ranges in between $\rho_0$ and $\rho_1$. Self-scheduling controller $K$ stabilizes the closed-loop system and minimizes the $H_{\infty}$ closed-loop performance from $d$ to $z$.

![Singular Values Controller at different Trim](image)

Fig. 4 Singular values of the interpolated controller at each operating point.

Affine parameter dependent controller is simply a convex combination of the two linear controllers at each vertex. As the parameter changes the dynamics of the parameter varying controller smoothly changes.

Hence, the resulting controller is in the form of

$$A_c(\rho) = A_{c0}\alpha + A_{c1}(1 - \alpha)$$
$$B_c(\rho) = B_{c0}\alpha + B_{c1}(1 - \alpha)$$
$$C_c(\rho) = C_{c0}\alpha + C_{c1}(1 - \alpha)$$
$$D_c(\rho) = D_{c0}\alpha + D_{c1}(1 - \alpha)$$

where the scheduling parameter can be given as $\alpha = 1 - \frac{\rho - \rho_0}{\rho_1 - \rho_0}$ and $\rho_0$ and $\rho_1$ are the end values of the parameter which form the two vertices of the controller. It is easy to observe that when the scheduling parameter $\rho$ equals to $\rho_0$, the controller is purely based on the linear controller which is synthesized for that vertex. On the other hand, when the scheduling parameter $\rho$ equals to $\rho_1$, the controller is purely based on the linear controller specific to that vertex. As $\rho$ varies in between $\rho_0$ and $\rho_1$, the controller will be a convex combination of both controllers, thus enabling a gain scheduling scheme. Singular values of the interpolated controller can be seen in Fig. 4.

More specifically, the equations can be rewritten as

$$x = A(\rho)x + B_d(\rho)d + B_u u$$
$$\dot{z} = C_x(\rho)x + D_{xd}d + D_{zu}u$$
$$y = C_y x$$

where $B_d = 0$, $B_u = 0$, $C_x = \begin{bmatrix} -C_{11} & 0 \end{bmatrix}$, $D_{xd} = \begin{bmatrix} I \end{bmatrix}$, $D_{zu} = \begin{bmatrix} 0 \end{bmatrix}$ and $C_y = C$ with proper dimensions in order to cast mixed sensitivity tracking problem as an $H_{\infty}$ optimization problem.
In this work, since an approximated affine parameter dependent model is available, it is easy to synthesize a gain scheduling $\mathcal{H}_\infty$ controller. A single affine parameter dependent controller can be used in order to control nonlinear helicopter model. The fundamental advantage of this affine parameter dependent controller is that the controller automatically adapts itself as the flight condition changes. However, it should be kept in mind that there is no guarantee that the parameter varying controller can stabilize nonlinear helicopter model. The success of this method largely based on the accuracy of linear parameter dependent helicopter model in capturing the nonlinear behavior of the helicopter model.

6. CLOSED-LOOP SYSTEM

Closed-loop system is a composition of parameter dependent model and parameter dependent controller as given below

$$\eta = A_{cl}\eta + B_{cl}d$$
$$z = C_{cl}\eta + D_{cl}d$$
$$y = C_{cl}y$$

(9)

Where $A_{cl} = \begin{bmatrix} A(\rho) & 0 \\ B_{cl}(\rho)C_y & A_c(\rho) \end{bmatrix}$, $B_{cl} = \begin{bmatrix} B_d \\ 0 \end{bmatrix}$, $C_{clx} = \begin{bmatrix} C_x + D_{zu}C_y \\ D_{zu}C_c(\rho) \end{bmatrix}$, $D_{cl} = D_{zd}$, $C_{cly} = \begin{bmatrix} C_y \\ 0 \end{bmatrix}$, and $\eta = \begin{bmatrix} x \\ z \end{bmatrix}$.

It is verified that the synthesized parameter dependent controller successfully stabilizes the closed-loop system simply by controlling the closed-loop eigenvalues at each operating point. Moreover, S and KS plots can be seen in Fig. 5 and Fig. 6.

For the sake of simplicity, we will be working on the longitudinal channel for the reference tracking purpose and the other three channels will be used for stabilization purpose only. Nevertheless, the controller synthesized can perform stabilization and tracking in all axes. As a simulation example, pilot demands a step change in the forward velocity and applies no commands at other axes. Parameter dependent controller will be performing reference tracking in the pitch channel and stabilization at all other axes.

6.1. Simulation Results on Parameter Dependent Model

Approximate linear parameter varying model is closed via a parameter dependent controller. 1 m/s forward velocity reference is applied at all trim conditions in order to verify that the controller successfully does stabilization and reference tracking. From Fig. 7 and Fig. 8, it is seen that the controller provides reference tracking following to a step reference command and the applied control input is reasonable.

Fig. 7. Response to a Forward Velocity Demand

Fig. 8. Lon. Cyc. Applied to a Forward Vel. Demand

6.2 Simulation Results at Nonlinear Model

There will be a single test case in order to ensure that the parameter varying controller successfully does stabilization and tracking. Helicopter is flying 10 m/s and the pilot demands 0 m/s.

Remark 1: Since the nonlinear model is vulnerable to rapid changes in the helicopter dynamics, 2 m/s rate limiting was applied to the reference command. Without reference filtration, large error signals are sent to the controller and large control signals are sent to the nonlinear model. Thus, the nonlinear closed-loop system exhibits largely different behavior than the closed-loop linear system. Notice that the nonlinear model is linearized via Jacobian linearization and
the linear model is expected to mimic the nonlinear model around small perturbations on the states and the controls. In order to overcome this problem, reference filtration has been done to prevent the rapid changes in the nonlinear model.

Create a linear parameter-varying controller that is suitable for such systems. An affine parameter-dependent controller synthesized for the Yamaha-R50 helicopter nonlinear model has proven to be successful. Future research directions include using a more sophisticated nonlinear model for the helicopter since the model that is built in this paper is quite simple. A better nonlinear model of the helicopter and addition of disturbances will make the results more reliable. Another problem is the size of the controller which is equal to the size of the augmented linear models. The order of the controller could be reduced for practical implementation.

7. REFERENCES


Balas, G. J., 2002. Linear Parameter Varying Control and its Application to Aerospace Systems. s.l., ICAS.


