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An Affine Parameter Dependent Controller of An Helicopter for Various Forward Velocity Conditions

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Abstract. In this paper, we address the design of a controller that accomplishes stabilization and reference tracking at various flight conditions by using linear helicopter models. The suggested controller is in the form of an H-infinity gain-scheduler, and is used for stabilization and reference tracking for the 4 axis(heave, pitch, roll and yaw) autopilot. Based on the linear models given for the Puma helicopter, an approximate affine parameter dependent model has been built. Then, a linear parameter dependent controller is synthesized which stabilizes the affine parameter dependent helicopter model. By doing so, a single controller achieves stabilization and reference tracking of a family of linear models by scheduling the controller gains based on the online measurement of the scheduling parameter, which is the forward velocity. Moreover, the affine parameter dependent controller is fitted into the linear models. It is observed that this single parameter dependent controller successfully stabilizes a family of linear helicopter model at different forward flight conditions.

Introduction

Control design for helicopters are generally based on classical design rules, and variations on the system dynamics are usually handled by the standard gain scheduling techniques by simply tuning the controllers at different flight conditions. In [1], a mixed sensitivity $\mathcal{H}_\infty$ controller is designed based on a linearized model of a Bell 205 helicopter as a design example. In [2], $\mathcal{H}_\infty$ controllers are designed using the linearizations extracted from nonlinear model of the Bell 205 helicopter.Moreover, in [3] an $\mathcal{H}_\infty$ flight control system is also designed to improve stability, maneuverability and agility for an helicopter. Resulting $\mathcal{H}_\infty$ flight control system is then fitted into the nonlinear model of a helicopter to simulate nonlinear dynamic response. In [4], a robust gain-scheduling algorithm is introduced by employing local multi-variable LQR controllers which are designed at each operation point, for a given performance index. In [5], a flight control system with gain scheduled LQ optimal controllers are designed based on look-up tables.

In the 90's Linear Parameter Varying (LPV) control has been presented as a reliable alternative to classical gain-scheduling for multivariable systems; typical well known examples can be found in [6] and [7]. Gain scheduling is a standard method to design controllers for dynamic systems over a wide performance envelope. In [8], LPV control design techniques are also applied to the attitude control of the X-33, a single-stage-to-orbit (SSTO) prototype vehicle. A multivariable LPV controller was designed using $\mathcal{H}_\infty$ synthesis for F/A-18 in [9].

In [10], two different approaches are proposed that can be used to obtain reliable LPV models. Unfortunately, there is no proposed way of obtaining quasi LPV models either by state transformation or function substitution for a nonlinear helicopter model. Generally, control designers use a family of linear, time-invariant (LTI) plants at different points of interest throughout the operational envelope in order to obtain a reliable LPV model. Based on the linear family of models, it is straightforward to construct a parameter dependent model. Hence, gain scheduling enables synthesis of global controllers based on interpolation of a family of locally linearized controllers.

In this paper, an affine parameter dependent controller is developed based on the family of linear helicopter models given in[11]. An affine parameter dependent system is obtained by using the family of linear plants. Moreover, the mixed sensitivity $\mathcal{H}_\infty$ controller synthesis approach is summarized. Based on the affine parameter dependent helicopter model, an affine parameter dependent controller is designed, which stabilizes the closed-loop system and provides reference tracking to the pilot reference commands in different flight conditions by scheduling the controller based on the online measurement of scheduling parameter. Finally, the parameter varying controller is tested at different linearization points.

Parameter Dependent Model

Linear helicopter model consists of 8 states, 4 inputs and 4 outputs which describe the internal dynamics, control inputs and outputs that are derived from the states [11]. The force and moment equations results in six equations with eight unknowns. Three longitudinal and three lateral equations are combined with two kinematic equations to describe the linear system.

A family of linear models can now be expressed as a parameter dependent model as

$$\begin{aligned} \dot{x} &\simeq A_p x + B_p u \\ y &= C_p x \end{aligned} \tag{1}$$

where $A_p = A_0 + A_1\rho$, $B_p = B_0 + B_1\rho$ and ρ is the forward velocity. The parameter dependent model is augmented by adding actuator dynamics where the actuator system matrices are A_{act} , B_{act}, and C_{act}.Hence the parameter dependent model with actuator dynamics can be given as

$$\begin{aligned} \dot{x} &= A(\rho)x + Bu \\ y &= Cx \end{aligned} \tag{2}$$

where $A(\rho) = \begin{bmatrix} A_p & B_p C_{act} \\ 0 & A_{act} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ B_{act} \end{bmatrix}$, $C = [C_p \quad 0]$ and $x = \begin{bmatrix} x_p \\ x_{act} \end{bmatrix}$.

Notice that the system matrix can be transformed into an affine parameter dependent system matrix $A(\rho) = \begin{bmatrix} A_0 + A_1\rho & (B_0+B_1\rho)C_{act} \\ 0 & A_{act} \end{bmatrix}$ and input matrix B is not parameterized by ρ . Moreover, the output is also derived from the states, so$C$ is also independent from parameter ρ. Indeed, addition of actuator dynamics transfers the parameter dependency on the input matrix to the system matrix.

Mixed Sensitivity $\mathcal{H}_\infty$controller Synthesis

The standard $\mathcal{H}_\infty$optimal control problem is equivalent to the minimization of the $\mathcal{H}_\infty$ norm over all stabilizing controllers. We consider the problem of synthesizing a controller K to satisfy attenuation of low frequency disturbances and minimization of high frequency actuator activity. In this formulation, the exogenous input signal d is the reference command r, and the controlled z variables are chosen to minimize low frequency error and high frequency control action. Details can be found in[1].

Gain Scheduled Controller

Aerospace control design rules based on classical control and gain scheduling is often employed since the dynamics of the air vehicle change as the flight conditions vary. However this gain scheduling method is simply based on designing a new controller at that flight condition or simply tuning the controller parameter for that specific flight condition. In this section, we will synthesize a self scheduling controller by using online measurement of scheduling parameter.

Given the affine parameter dependent system P,

$$\begin{aligned} \dot{x} &= A(\rho)x + B_d(\rho)d + B_u u \\ z &= C_z(\rho)x + D_{zd}(\rho)d + D_{zu}u \end{aligned} \tag{3}$$

$$y = C_y x + D_{yd} d + D_{yu} u$$

controlled by an affine parameter dependent controller K,

$$\begin{aligned} x_c &= A_c(\rho)x_c + B_c(\rho)\, y \\ u &= C_c(\rho)x_c + D_c(\rho)y \end{aligned} \tag{4}$$

which is scheduled by the online measurement of parameter ρ where ρ ranges in between ρ_0 and ρ_1. Self scheduling controller K stabilizes the closed-loop system and minimizes the $\mathcal{H}_\infty$ closed-loop performance from d to z.Affine parameter dependent controller is simply a convex combination of the two linear controllers at each vertex. As the parameter changes, the dynamics of the parameter varying controller smoothly changes. Hence, the resulting controller is given in the form of

$$\begin{aligned} A_c(\rho) &= A_{c0}\alpha + A_{c1}(1-\alpha) \\ B_c(\rho) &= B_{c0}\alpha + B_{c1}(1-\alpha) \\ C_c(\rho) &= C_{c0}\alpha + C_{c1}(1-\alpha) \\ D_c(\rho) &= D_{c0}\alpha + D_{c1}(1-\alpha) \end{aligned} \tag{5}$$

where the scheduling parameter can be given as $\alpha = 1 - \frac{\rho-\rho_0}{\rho_1-\rho_0}$ and ρ_0, ρ_1 are the end values of the parameter and form two vertices of the controller. More specifically, the equations can be rewritten as

$$\begin{aligned} x &= A(\rho)x + B_d d + B_u u \\ z &= C_z x + D_{zd} d + D_{zu}\, u \\ y &= C_y x \end{aligned} \tag{6}$$

where $B_d = 0$, $B_u = B$, $C_z = \begin{bmatrix} -C \\ 0 \end{bmatrix}$, $D_{zd} = \begin{bmatrix} I \\ 0 \end{bmatrix}$, $D_{zu} = \begin{bmatrix} 0 \\ I \end{bmatrix}andC_y = C$ with proper dimensions in order to cast mixed sensitivity tracking problem as an $\mathcal{H}_\infty$ optimization problem.

In this work, since an approximated affine parameter dependent model is available, it is easy to synthesize a gain scheduling $\mathcal{H}_\infty$ controller. A single affine parameter dependent controller can be used in order to control the family of linear helicopter models. The fundamental advantage of this affine parameter dependent controller is that the controller automatically adjusts itself as the flight condition changes. However, it should be noted that there is no guarantee that the parameter varying controller can stabilize the linear helicopter model family. The success of this method is largely based on the accuracy of linear parameter dependent helicopter model in capturing the behavior of the helicopter model family. If the parameter dependent model is sufficiently close to the model family, synthesized linear parameter dependent model successfully stabilizes the helicopter model.

Closed-Loop System

Closed-loop system is a composition of parameter dependent model and parameter dependent controller as given below

$$\begin{aligned} \eta &= A_{cl}\eta + B_{cld}\, d \\ z &= C_{clz}\eta + D_{cl} d \\ y &= C_{cly}\eta \end{aligned} \tag{7}$$

where $A_{cl} = \begin{bmatrix} A(\rho) & 0 \\ B_c(\rho)C_y & A_c(\rho) \end{bmatrix}$, $B_{cld} = \begin{bmatrix} B_d \\ 0 \end{bmatrix}$,$C_{clz} = [C_z + D_{zu}D_c(\rho)C_y \quad D_{zu}C_c(\rho)]$, $D_{cl} = D_{zd}$, $C_{cly} = [C_y \quad 0]$, and$\eta = \begin{bmatrix} x \\ x_c \end{bmatrix}$.

It is verified that the synthesized parameter dependent controller successfully stabilizes the closed-loop system simply by controlling the closed-loop eigenvalues at each operating point.

For the sake of simplicity, we will be working on the longitudinal channel for the reference tracking purpose and the other three channels will be used for stabilization purpose only. Nevertheless, the controller synthesized can perform stabilization and tracking in all axes. As a simulation example, pilot demands a step change in the forward velocity and applies no commands at other axes.

Simulation Results on Parameter Dependent Model. Approximate linear parameter varying model is closed via a parameter dependent controller. 1 knot forward velocity reference is applied at all trim conditions in order to verify that the controller successfully does stabilization and reference tracking. From Fig. 1, it is seen that the controller provides reference tracking following to a step reference command and the applied control inputs are reasonable.

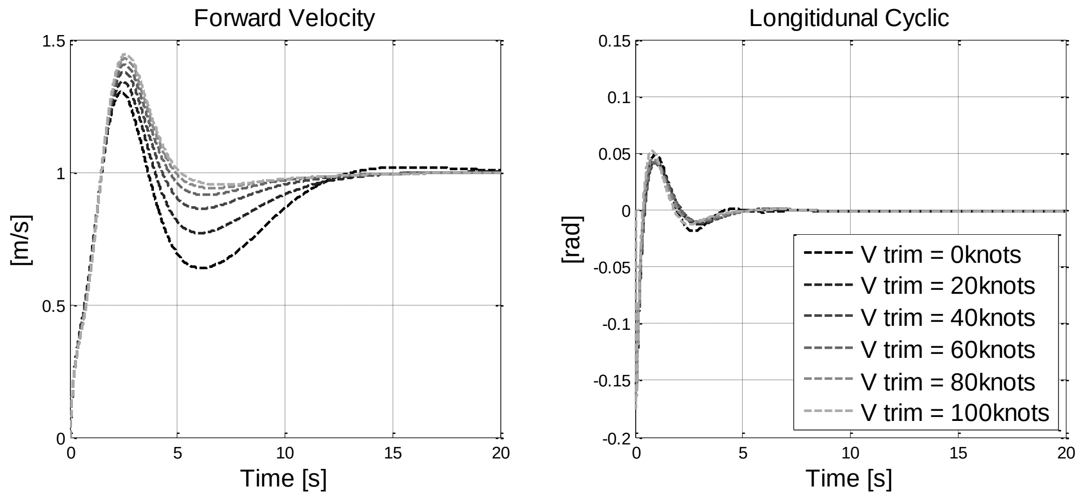

Fig. 1. On axis Response to a Forward Velocity Demand (Longitudinal Axis)

It is also interesting to observe responses at other axes. In Fig. 2, Fig. 3 and Fig. 4, it is observed that stabilization is accomplished all other axes. The oscillations are simply due to the cross coupled nature of the helicopter dynamics.

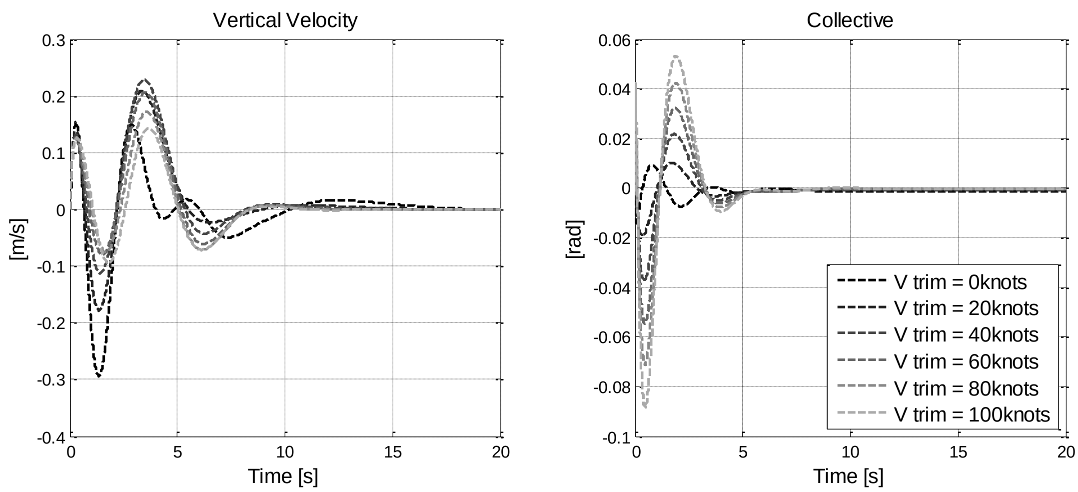

Fig. 2.Off axis response to a Forward Velocity Demand (Heave axis)

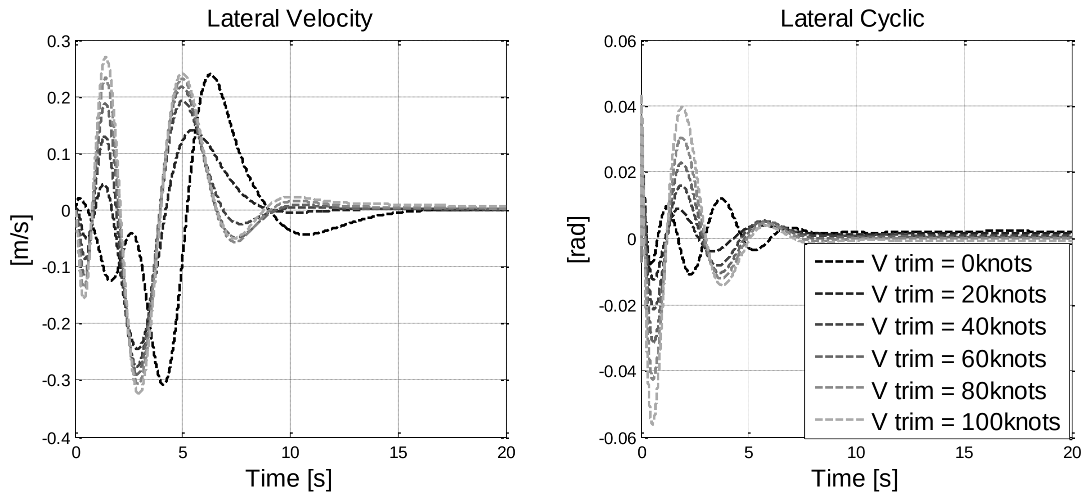

Fig. 3. Off axis response to a Forward Velocity Demand (Lateral axis)

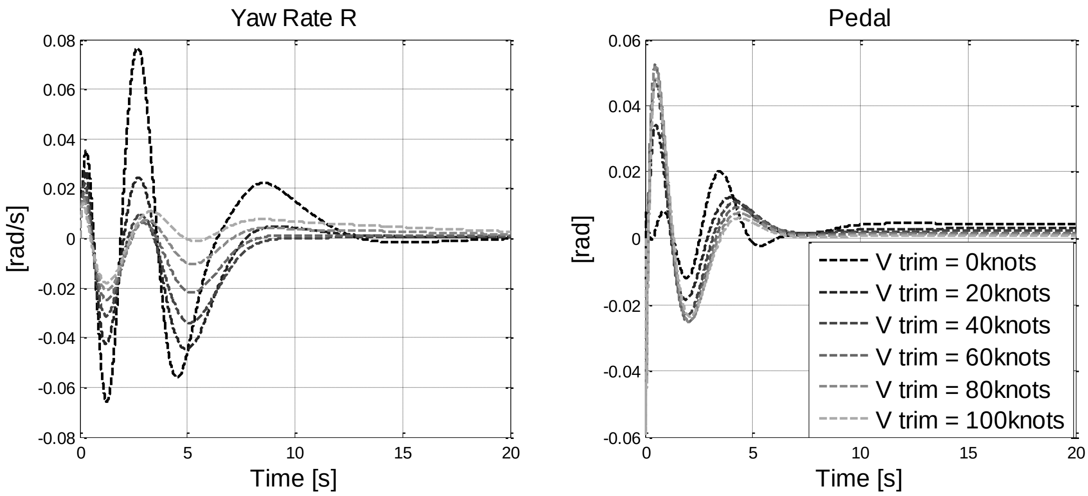

Fig. 4.Off axis response to a Forward Velocity Demand (Yaw axis)

Simulation Results on Different Linear Models. The parameter dependent linear controller is also fitted into the linear models at all trim conditions. Trimmed linear model is closed via the parameter dependent controller, then 1 knot forward velocity step reference is applied at all trim conditions. The parameter dependent controller successfully accomplishes tracking of on axis plot commands and stabilization on all other axes. Response to a step forward velocity command and longitudinal cyclic applied at all trim conditions can be seen in Fig. 5. The off axis responses have not been included due to space limitations, however, it is observed that stabilization is achieved after some oscillations, similar to the parameter dependent model case.

Summary

Aerospace systems have very extensive operating domains where change in the dynamics need to be handled with a scheduling technique. This paper has introduced a parameter dependent control methodology, which uses mixed sensitivity $\mathcal{H}_\infty$ synthesis to create a linear parameter-varying controller that is suitable for such systems. An affine parameter dependent controller synthesized for the linear helicopter models for Puma has proven to be successful. Future research directions include using a nonlinear model for the helicopter since the models that are used for the simulation purposes are all linear. In future studies, a nonlinear math model will be constructed in order to see the nonlocal performance of the designed parameter dependent controller.

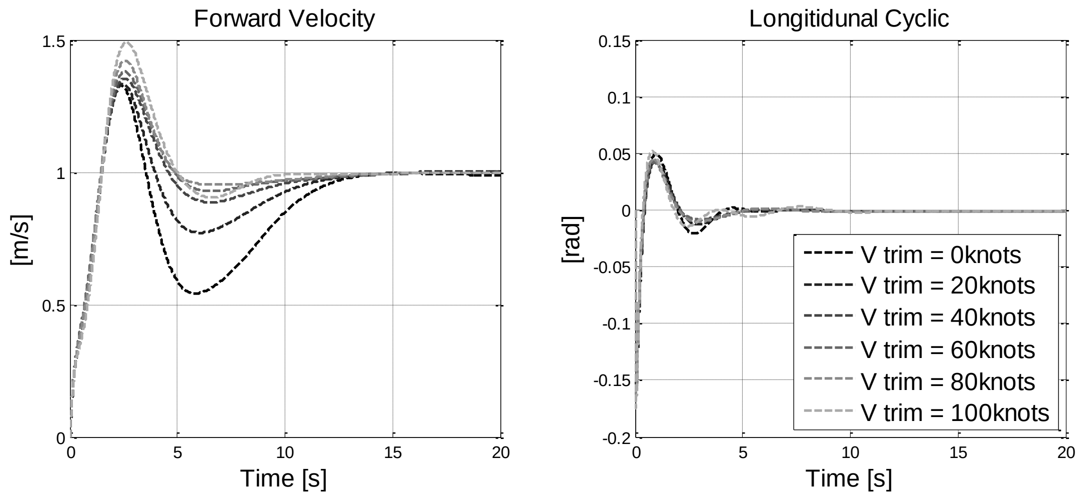

Fig. 5.On axis Response to a Forward Velocity Demand (Longitudinal Axis)

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