17th Mediterranean Conference on Control & Automation
Makedonia Palace, Thessaloniki, Greece
June 24 - 26, 2009

Frequency Constrained Control of Oscillations in Flow Problems

Coşku Kasnakoğlu
Department of Electrical and Electronics Engineering
TOBB University of Economics and Technology
Söğütözü, Ankara 06560, Turkey
Email: kasnakoglu@etu.edu.tr

***Abstract*—In this paper we investigate the control of flow problems where the control objective is to reduce the oscillation amplitude while keeping the frequency of oscillation between predefined limits at all times. Starting from a simple model representing the oscillatory mode dynamics of the governing equations, the conditions that the control parameters must satisfy in order to achieve the desired objective are derived in detail. The results obtained are illustrated on a physical application example, namely cavity flow control, where it is seen that the controller is successful in achieving the control goal.**

I. INTRODUCTION

The motion of liquids and gases is a phenomenon that one encounters continuously in everyday life. The flow of air around the body of a car or the wing of an aircraft, the motion of petroleum through pipelines, flow of water in oceans and the motion of air in the atmosphere carrying the clouds are a few examples of such flows. Flow control refers to the ability to manipulate fluid flow in order to achieve a desired change in its behavior. Flow control is a challenging interdisciplinary topic and offers many potential technological benefits, such as reducing fuel costs for land, air and sea vehicles, and improving effectiveness of industrial processes [1, 2]. Among myriad of research on the topic one can find studies on the control of channel flows [3], stabilization of bluff-body flow [4], control of cylinder wakes [5, 6], control of cavity flows [7, 8] and optimal control of vortex shedding [9].

In this paper we study the suppression of unwanted oscillations in fluid flow problems, utilizing a model obtained by the aforementioned reduction strategies. The control design is subject to the constraint that the frequency of oscillation must be kept between certain limits at all times. Such a constraint is important in real life problems for various reasons. For instance, many actuators are unable to produce excitations and many transducers are unable to take measurements outside a certain frequency range. An example is synthetic jet actuators, which are of the most commonly used actuators in flow control [10, 11]. Synthetic jet actuators operate by moving a membrane or diaphragm up and down, sucking the surrounding fluid into a chamber and then expelling it. The lack of an external source for the fluid makes it necessary that the device keep vibrating to sustain its operation. Hence the flow must remain oscillatory at all times so that a feedback controller can be implemented through these devices. Similar arguments can be made for the transducers used for measurements. It is therefore of practical importance to obtain a control design that will be able to reduce the amplitude of the unwanted oscillation, while at the same keeping the oscillation frequency within desired limits, which is the main topic of this paper.

The paper is organized as follows: Section II introduces and describes the problem. Section III presents the main results, which are the conditions under which the oscillation amplitude can be suppressed while maintaining the frequency within a desired range at all times. Section IV demonstrates the application of the results to a real-life flow control problem, namely the control of unwanted oscillations resulting from air flow past a cavity. Section V concludes the paper with final discussions and future work ideas.

II. PROBLEM DESCRIPTION

Fluid flow processes are most commonly described by the Navier-Stokes partial differential equations (PDEs). We shall consider the flow to be isentropic to simplify the final form of the system. With this treatment, it was shown in Rowley et al. [12], that the compressible Navier-Stokes equations can be written as[1]

$$\frac{D\boldsymbol{u}}{Dt} + \frac{1}{M^2}\frac{2}{\kappa-1}\nabla c = \frac{1}{\mathrm{Re}}\nabla^2\boldsymbol{u} \tag{1}$$

$$\frac{Dc}{Dt} + \frac{\kappa-1}{2}\,c\,\mathrm{div}\,\boldsymbol{u} = 0 \tag{2}$$

where $\boldsymbol{u}(\boldsymbol{x},t) = (u_{\mathrm{s}}(\boldsymbol{x},t), u_{\mathrm{n}}(\boldsymbol{x},t))$ is the flow velocity in the stream-wise and normal direction, $c(\boldsymbol{x},t)$ is the local speed of sound, the operator $D/Dt = \partial/\partial t + \boldsymbol{u}\cdot\nabla$ stands for the material derivative, and $\boldsymbol{x} = (x,y)$ denotes Cartesian coordinates over the spatial domain $\Omega \subset \mathbb{R}^2$. The constants κ, Re, and M denote respectively the ratio of specific heats, Reynolds number, and Mach number. The Navier-Stokes equations (1)-(2) are subject to some initial conditions and boundary conditions, and the system input $\gamma \in \mathbb{R} \to \mathbb{R}$ affects the system through the latter. Also let $\mathbb{H} = \mathcal{L}_2(\Omega, \mathbb{R}^3)$ be the square-integrable functions on Ω, where $\boldsymbol{q} := (u_{\mathrm{s}} - u_{\mathrm{s}0}, u_{\mathrm{n}} - u_{\mathrm{n}0}, c - c_0) \in \mathbb{H}$ is the fluctuation of the flow velocity about the mean value

[1] These equations have been non-dimensionalized by scaling $\boldsymbol{u}$ by the freestream velocity U_∞, the local speed of sound by the ambient sound speed $c_\infty = (\kappa R T_\infty)^{1/2}$, where T_∞ is the ambient temperature, the cartesian coordinates $\boldsymbol{x}$ by the cavity depth D, time by D/U_∞, and pressure by $\bar{\rho}U_\infty^2$, where $\bar{\rho}$ denotes mean density.

978-1-4244-4685-8/09/$25.00 ©2009 IEEE

$\boldsymbol{q}_0 = (u_{n0}, u_{s0}, c_0)$. In this paper we shall focus our attention to flows that exhibit an undesired oscillation in the absence of a control action. Examples of such flows include the flow over a shallow cavity [8, 10], and the flow around a circular cylinder [5, 6]. The goal is to find control law γ so that the oscillation amplitude is reduced, while the frequency of oscillation is kept between desired limits at all times.

To perform the design directly on the Navier-Stokes equations (1)-(2) is very difficult, if not impossible, due to the complicated and infinite dimensional nature of these PDEs. Therefore we shall first look for a means to simplify these equations. A common approach for the simplification of the Navier-Stokes equations is to employ proper orthogonal decomposition (POD) followed by Galerkin Projection (GP) [13, 14]. In this approach one first obtains a set of POD modes for this system denoted by $\{q_i(x)\}_{i=0}^N$. These POD modes are orthonormal i.e. $(q_i, q_j)_\Omega = \delta_{ij}$, where the inner product is defined as $(u, v) := \int_\Omega u \cdot v \, dV$. Projecting the velocity vector $\boldsymbol{q}$ onto these modes one obtains the POD expansion as:

$$\boldsymbol{q}(x,t) \approx \boldsymbol{q}^{[N]}(x,t) = \boldsymbol{q}_0(x) + \sum_{i=1}^{N} a_i(t) q_i(x) \; . \tag{3}$$

The coefficients $a_i(t)$ are called *POD coefficients* and they capture the time dependence. Equation (3) is then substituted into (1)-(2) to obtain the dynamics in terms of the time coefficients $\{a_i(t)\}_{i=0}^N$. Following a shift by the equilibrium point, this procedure yields a set of N differential equations

$$\dot{a}_i = \frac{1}{Re} \sum_{j=1}^{N} l_{ij} a_j + \sum_{j,k=1}^{N} q_{ijk} a_j a_k \tag{4}$$

where l_{ij}, q_{ijk} are the Galerkin system coefficients. System (4) can be expressed in compact form as

$$\dot{a} = La + Q(a,a) \tag{5}$$

where $a = \{a_i\}_{i=1}^N \in \mathbb{R}^N$, $L = \{l_{ij}\}_{i,j=1}^N \in \mathbb{R}^{N \times N}$, $Q(a) = \{a^T Q_i a\}_{i=1}^N \in \mathbb{R}^N$, $Q_i = \{q_{ijk}\}_{j,k=1}^N \in \mathbb{R}^{N \times N}$. Note that the effect of the system input γ is not directly visible (4) and (5) as it gets embedded into the Galerkin system coefficients l_{ij} and q_{ijk}. Techniques for input separation (IS) such as those in Camphouse [15], Efe and Ozbay [16], Kasnakoglu et al. [17] have been devised to remedy this situation, which make the control input γ appear explicitly through linear, quadratic and/or bilinear terms in the Galerkin system equations. We shall consider the case in which the input terms are linear to simplify the analysis, which yields to a Galerkin system of the form

$$\dot{a} = La + Q(a,a) + B\gamma \tag{6}$$

where $B = \{b_i\}_{i=1}^N \in \mathbb{R}^N$. The Galerkin system can be further simplified using a Kryloff-Bogoliubov (KB) ansatz [5, 18], which is of the form $a_1 = r\cos\omega t$, $a_2 = r\sin\omega t$, and $a_i = k_i$ for $i \geq 3$. Here, a_1, a_2 are the *oscillatory modes*, a_i for $i \geq 3$ are the *shift modes*, $r^2 := a_1^2 + a_2^2$, $\theta := \arctan(a_2/a_1) = \omega t$, and $k_i \in \mathbb{R}_+$. Utilizing mean field models, invariant manifold reduction or center manifold theory, it can be shown that the shift-mode amplitudes k_i are locally slaved to the oscillation amplitude r [5, 19]. Using this dependence to eliminate the shift modes from the equations, and utilizing the KB ansatz, one arrives at the simplified evolution equations for the oscillatory modes as

$$\frac{\partial a_1}{\partial t} = \sigma a_1 - \omega a_2 - \alpha a_1^3 - \alpha a_2^2 + b_1 \gamma \tag{7}$$

$$\frac{\partial a_2}{\partial t} = \omega a_1 + \sigma a_2 - \alpha a_1^2 - \alpha a_2^3 + b_2 \gamma \tag{8}$$

where $\sigma, \alpha, \omega \in R_+$ are functions of the Galerkin system coefficients l_{ij} and q_{ijk}. Expressed in polar coordinates the system (r, θ), the system (7)-(8) becomes

$$\dot{r} = \sigma r - \alpha r^3 + (b_1 \cos(\theta) + b_2 \sin(\theta))\gamma \tag{9}$$

$$\dot{\theta} = \omega + \frac{1}{r}(b_2 \cos(\theta) - b_1 \sin(\theta))\gamma \; . \tag{10}$$

A natural choice for the control law γ in oscillatory fluid flow problems is to apply a sinusoidal signal whose amplitude and phase are varied based on the system states, i.e.

$$\gamma(t) = Ar(t)\cos(\theta(t) - \phi) \tag{11}$$

where $A \in \mathbb{R}$, $\phi \in [0, 2\pi]$ are controller parameters to be determined.[2] Substituting (11) into (9)-(10) yields

$$\dot{r} = (\sigma + A(\cos(\theta)\, b_1 + \sin(\theta)\, b_2) \times (\cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)))\, r - \alpha\, r^3 \tag{12}$$

$$\dot{\theta} = \omega + A(\cos(\theta)\, b_2 - \sin(\theta)\, b_1) \times (\cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)) \; . \tag{13}$$

The goal is to determine the control parameters A and ϕ such that the following two criteria are satisfied:

Criterion 1 (Frequency criterion). The oscillation frequency remains within predefined design limits at all times, i.e. given $\omega_{\rm low}$, $\omega_{\rm high}$ such that $0 < \omega_{\rm low} < \omega < \omega_{\rm high}$, we require that $\omega_{\rm low} < \dot{\theta} < \omega_{\rm high}$ for all $t > 0$.

Criterion 2 (Amplitude criterion). The oscillation amplitude tends to zero with time, i.e. $r \to 0$ as $t \to \infty$.

In the next section we will derive guidelines as to how the parameters A and ϕ can be selected to satisfy these criteria.

III. Main Results

In this section we present the results regarding the selection of the control parameters so as to satisfy the design goals given in Criteria 1 and 2.

Theorem 3 (Frequency condition)**.** *For the fluid flow problem described in Section II, let $\omega_{\rm low} \in \mathbb{R}_+$ and $\omega_{\rm high} \in \mathbb{R}_+$ be the lower and upper bounds of the allowable frequency range of oscillation, where $0 < \omega_{\rm low} < \omega < \omega_{\rm high}$. A control law of the form $\gamma = A^* r\cos(\theta - \phi^*)$ will achieve*

$$\omega_{\rm low} < \dot{\theta} < \omega_{\rm high} \quad \forall t > 0 \tag{14}$$

[2] A similar model and controller structure was used in [20] for the control of cavity flows.

if (ϕ^*, A^*) *is an element of* $\mathcal{S}_f \subset \mathbb{R}^2$*, defined as*

$$\mathcal{S}_f := \mathcal{S}_1 \cap \mathcal{S}_2 \quad (15)$$

where

$$\mathcal{S}_1 := \{(\phi, A) : A_1(\phi) < A < A_2(\phi)\} \quad (16)$$
$$\mathcal{S}_2 := \{(\phi, A) : A_1'(\phi) < A < A_2'(\phi)\} \quad (17)$$

and

$$A_1(\phi) = \frac{2\,(\omega-\omega_{\text{low}})\left(b_2\cos(\phi)-b_1\sin(\phi)-\sqrt{{b_2}^2+{b_1}^2}\right)}{(b_1\cos(\phi)+b_2\sin(\phi))^2} \quad (18)$$
$$A_2(\phi) = \frac{2\,(\omega-\omega_{\text{low}})\left(b_2\cos(\phi)-b_1\sin(\phi)+\sqrt{{b_2}^2+{b_1}^2}\right)}{(b_1\cos(\phi)+b_2\sin(\phi))^2} \quad (19)$$
$$A_1'(\phi) = \frac{2\,(\omega-\omega_{\text{high}})\left(b_2\cos(\phi)-b_1\sin(\phi)+\sqrt{{b_2}^2+{b_1}^2}\right)}{(b_1\cos(\phi)+b_2\sin(\phi))^2} \quad (20)$$
$$A_2'(\phi) = \frac{2\,(\omega-\omega_{\text{high}})\left(b_2\cos(\phi)-b_1\sin(\phi)-\sqrt{{b_2}^2+{b_1}^2}\right)}{(b_1\cos(\phi)+b_2\sin(\phi))^2}\ . \quad (21)$$

Proof: Let us first determine the conditions under which $\dot\theta > \omega_{\text{low}}$. Using the dynamics of θ from (12)-(13), we can write

$$\omega_{\text{low}} < \dot\theta = \omega + A\left(\cos(\theta)\,b_2 - \sin(\theta)\,b_1\right)\times \left(\cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)\right)\ .$$

Dividing both sides by $\cos^2(\theta)$, using the identity $\cos^{-2}(\theta) = \sec^2(\theta) = 1+\tan^2(\theta)$, expanding and collecting terms yields

$$0 < (\omega - \omega_{\text{low}} - Ab_1\sin(\phi))\tan^2(\theta) + (Ab_2\sin(\phi) - Ab_1\cos(\phi))\tan(\theta) + Ab_2\cos(\phi) + \omega - \omega_{\text{low}}\ .$$

The right hand side of this equation is a quadratic polynomial in $\tan(\theta)$, which must always be positive for the inequality above to be satisfied. This can be achieved if the leading coefficient is positive and the discriminant Δ_1 is negative. The former requires that

$$(\omega - \omega_{\text{low}} - Ab_1\sin(\phi)) > 0\ . \quad (22)$$

For the latter, the negativity condition of the discriminant Δ_1 can be written as

$$\Delta_1 = (b_2\sin(\phi) + b_1\cos(\phi))^2 A^2 + (-4(\omega-\omega_{\text{low}})b_2\cos(\phi) + 4b_1\sin(\phi)(\omega - \omega_{\text{low}}))\,A - 4(\omega-\omega_{\text{low}})^2 < 0\ .$$

The equation above for Δ_1 is quadratic in A with a positive leading coefficient $(b_2\sin(\phi)+b_1\cos(\phi))^2$. Therefore it will be negative between its roots, which can be computed from the quadratic formula to be of the form given by A_1 and A_2 in (18) and (19). Therefore for a given (A^*, ϕ^*), we will have $\dot\theta > \omega_{\text{low}}$ if (A^*, ϕ^*) satisfies (22), as well as $A_1(\phi^*) < A^* < A_2(\phi^*)$. If we let

$$\mathcal{S}_0 := \{(\phi, A) : \omega - \omega_{\text{low}} - Ab_1\sin(\phi) > 0\}$$
$$\mathcal{S}_1 := \{(\phi, A) : A_1(\phi) < A < A_2(\phi)\}$$

then this equivalent to saying that $(\phi^*, A^*) \in \mathcal{S}_0 \cap \mathcal{S}_1$. It can also be shown that $\mathcal{S}_1 \subset \mathcal{S}_0$ so the condition for $\dot\theta > \omega_{\text{low}}$ becomes $(\phi^*, A^*) \in \mathcal{S}_1$. The conditions for $\dot\theta < \omega_{\text{high}}$ can be obtained similarly to be $(A^*, \phi^*) \in \mathcal{S}_0' \cap \mathcal{S}_2$, where

$$\mathcal{S}_0' := \{(\phi, A) : \omega - \omega_{\text{high}} - Ab_1\sin(\phi) < 0\}$$
$$\mathcal{S}_2 := \{(\phi, A) : A_1'(\phi) < A < A_2'(\phi)\}$$

and A_1', A_2' are as given in (20) and (21). In addition, it can be proved that $\mathcal{S}_2 \subset \mathcal{S}_0'$ so the condition for $\dot\theta < \omega_{\text{high}}$ becomes $(A^*, \phi) \in \mathcal{S}_2$.[3] Collecting the results for $\dot\theta > \omega_{\text{low}}$ and $\dot\theta < \omega_{\text{high}}$ cases above, the condition for the frequency criterion becomes $(\phi^*, A^*) \in \mathcal{S}_1 \cap \mathcal{S}_2$, which is the statement of the theorem. ∎

Theorem 4 (Magnitude condition)**.** *For the fluid flow problem described in Section II, a control law of the form* $\gamma = A^* r\cos(\theta - \phi^*)$ *where* $(\phi^*, A^*) \in \mathcal{S}_f$ *with* $\mathcal{S}_f$ *is as given in (15), will achieve*

$$r \to 0 \quad \text{as } t \to \infty \quad (23)$$

if

$$(\phi^*, A^*) \in \mathcal{S}_m \quad (24)$$

where

$$\mathcal{S}_m := \left\{(\phi, A) : \tfrac{1}{2}Ab_1\cos(\phi) + \sigma + \tfrac{1}{2}Ab_2\sin(\phi) < 0\right\}\ . \quad (25)$$

Proof: We first rewrite the controlled system in polar form given in (12)-(13) compactly as follows

$$\dot r = g(\theta, \phi, A)\,r - \alpha\,r^3 \quad (26)$$
$$\dot\theta = h(\theta, \phi, A) \quad (27)$$

where

$$g(\theta, \phi, A) := \sigma + (\cos(\theta)\,b_1 + \sin(\theta)\,b_2)\times A\left(\cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)\right)$$
$$h(\theta, \phi, A) := \omega + (\cos(\theta)\,b_2 - \sin(\theta)\,b_1)\times A\left(\cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)\right)\ .$$

Since the control parameters A and ϕ are selected such that $(\phi, A) \in \mathcal{S}$, we have $0 < \omega_{\text{low}} < \dot\theta < \omega_{\text{high}}$, so $\dot\theta$ is always positive, which means that θ is strictly increasing in time. Therefore, one can map time values t to angle values θ with a one to one and onto function χ such that

$$\theta = \chi(t) \text{ and } t = \chi^{-1}(\theta) \quad (28)$$

where χ is the solution of (27) starting from $\theta(0) = \theta_0$. This one to one and onto correspondence makes in meaningful to view θ as a new time scale instead of t, and investigate the dynamics of r with respect to θ, which is

$$\frac{dr}{d\theta} = \frac{dr}{dt}\frac{dt}{d\theta} = \frac{\dot r}{\dot\theta} = \frac{\left(g(\theta,\phi,A)\,r - \alpha\,r^3\right)}{\dot\theta}\ .$$

Since $0 < \omega_{\text{low}} < \dot\theta < \omega_{\text{high}}$ one can write

$$\frac{g(\theta,\phi,A)\,r - \alpha\,r^3}{\omega_{\text{high}}} < \frac{dr}{d\theta} < \frac{g(\theta,\phi,A)\,r - \alpha\,r^3}{\omega_{\text{low}}}\ .$$

[3]The proof that $\mathcal{S}_1 \subset \mathcal{S}_0$ and $\mathcal{S}_2 \subset \mathcal{S}_0'$ has been omitted due to space limitations.

It therefore makes sense to analyze the following two systems

$$\Sigma_1 \ : \quad \frac{dr}{d\theta} = \omega_{\text{high}}^{-1} \left(g\left(\theta,\phi,A\right) r - \alpha \, r^3 \right) \qquad (29)$$

$$\Sigma_2 \ : \quad \frac{dr}{d\theta} = \omega_{\text{low}}^{-1} \left(g\left(\theta,\phi,A\right) r - \alpha \, r^3 \right) \qquad (30)$$

as they provide bounds for the behavior of the original system. Let us start with system Σ_1 in (29), divide by r^3, define a change of variables $s := 1/r^2$ and differentiate to get

$$-\frac{1}{2}\frac{ds}{d\theta} = \omega_{\text{high}}^{-1} \left(g\left(\theta,\phi,A\right) s - \alpha \right) \ . \qquad (31)$$

Defining

$$M(\theta,\phi,A) := e^{2\omega_{\text{high}}^{-1} \int_0^\theta g(z,\phi,A)dz} \ ,$$

multiplying both sides of (31) by $M(\theta)$, rearranging and collecting terms yields

$$\frac{d}{d\theta}\left(sM(\theta,\phi,A)\right) = 2\omega_{\text{high}}^{-1} M(\theta,\phi,A)\alpha \ .$$

Integrating both sides, transforming from s back to r, rearranging and solving for r yields

$$r(\theta)^2 = \frac{M(\theta,\phi,A) r_0^2}{2\omega_{\text{high}}^{-1} r_0^2 \alpha \int_0^\theta M(y,\phi,A)dy + 1} \ . \qquad (32)$$

If the control parameters are selected such that $(\phi^*, A^*) \in \mathcal{S}_m$, then it can be shown that as $\theta \to \infty$, $M(\theta,\phi^*,A^*) \to 0$ and $0 < C_4 \le \lim_{\theta\to\infty} \int_0^\theta M(y,\phi,A)dy \le C_5$ for some $C_4, C_5 \in \mathbb{R}_+$.[4] Therefore we see from (32) that $r \to 0$ as $\theta \to \infty$. The same result can be obtained for system Σ_2 in (30) in a similar fashion. Since the trajectories of Σ_1 and Σ_2 bound the trajectories of the original system from above and below, it follows that $r \to 0$ as $\theta \to \infty$ for the original system as well. Since $\theta = \chi(t)$ where χ is monotonically increasing, we conclude that $r \to 0$ as $t \to \infty$, which is the statement of the theorem. ∎

IV. Application Example: Cavity Flow Control

In this section a physical flow control problem, namely the suppression of unwanted oscillations generated by the air flow over a shallow cavity is considered as an example. As mentioned in the introduction, this is a problem that has captured significant research interest [7, 8, 10, 12], and has been the initial motivation for this study. Air flowing over a shallow cavity exhibits a strong self-sustained resonance caused by a natural feedback mechanism. Acoustic waves are scattered by shear layer structures impacting the trailing edge of the cavity. These acoustic waves travel upstream to reach the receptivity region of the shear layer, where they tune and enhance the development and growth of shear layer structures. The resulting acoustic fluctuations can be very intense and are known to cause, among other problems, store damage and airframe structural fatigue in weapons bay applications. To suppress or reduce the pressure fluctuations inside the cavity, feedback control is applied to the flow by using a synthetic jet-like actuator, which is typically an acoustic actuator located at the cavity trailing edge [8]. A schematic representation is illustrated in Figure 1.

[4]The proof of this statement has been omitted due to space limitations.

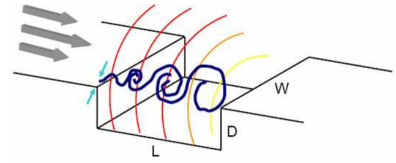

Fig. 1. Control of cavity flow oscillations using actuation at the cavity trailing edge. (figure courtesy of OSU GDTL)

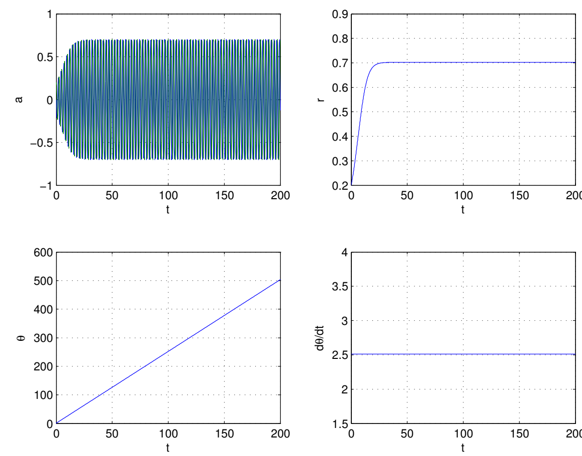

Fig. 2. Unforced response of the oscillatory mode dynamics.

As described in Section II, the first step in the control design process is to start with the Navier-Stokes PDEs governing the flow process and obtain a Galerkin system describing the flow. For this purpose we utilize the system parameters from the study by Samimy et al. [8] and take $N = 20$ for the order of the Galerkin system. This allows the Galerkin system to capture a sufficient amount of energy to produce a faithful representation of the flow. Proceeding with further reduction utilizing a KB ansatz we arrive at the oscillatory mode dynamics (9)-(10), with the parameter values given as $\sigma = 0.1368$, $\alpha = 0.2790$, $\omega = 2.5133$, $b_1 = -7.6412 \times 10^{-4}$, $b_2 = -1.1000 \times 10^{-4}$. The unforced response of the oscillation mode dynamics is given in Figure 2 and the response of the higher order Galerkin model is given in Figure 3 for comparison. In Figure 3, only the first four modes of the Galerkin system are plotted as plotting all 20 would produce clutter. It is seen that the oscillatory mode dynamics adequately represent key trends and qualities of the response, such as the rise time, oscillation amplitude and oscillation frequency, and the quantitative values produced by the simple model are also reasonable. For the control design, we set the frequency limits to be $\omega_{\text{low}} = 2.14$ and $\omega_{\text{high}} = 2.89$, which allows a frequency change of about 15% from ω in both directions. The curves

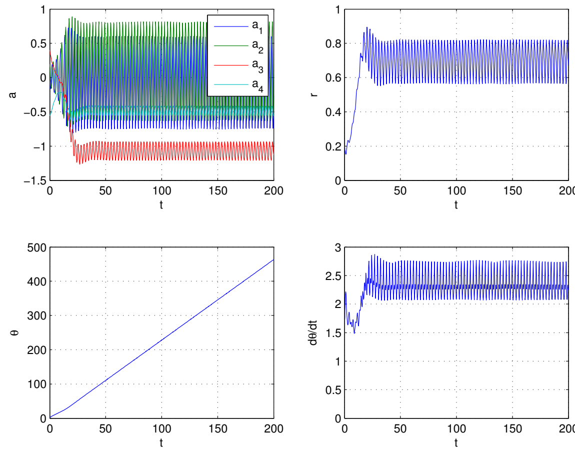

Fig. 3. Unforced response of the high order Galerkin model.

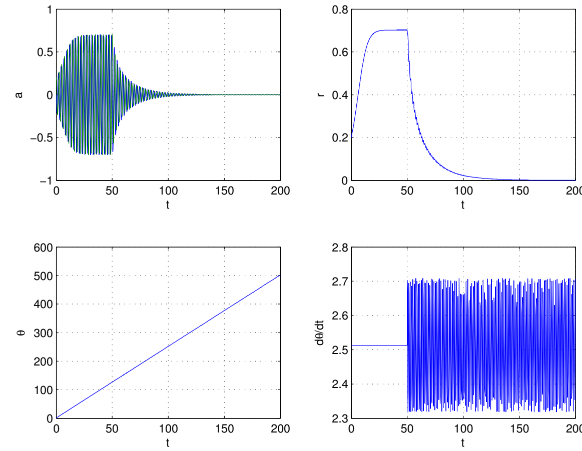

Fig. 5. Controlled response of the oscillatory modes.

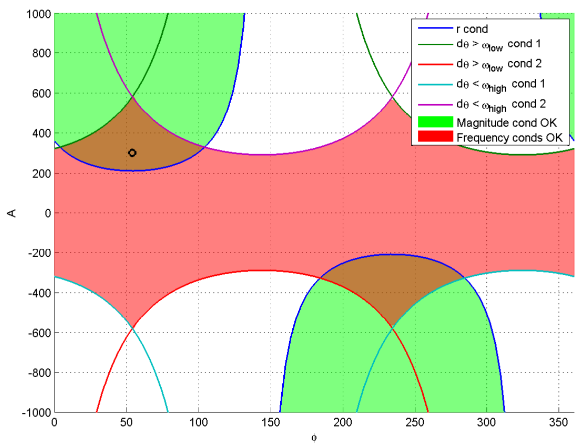

Fig. 4. Frequency and magnitude conditions for controller parameter values A and ϕ.

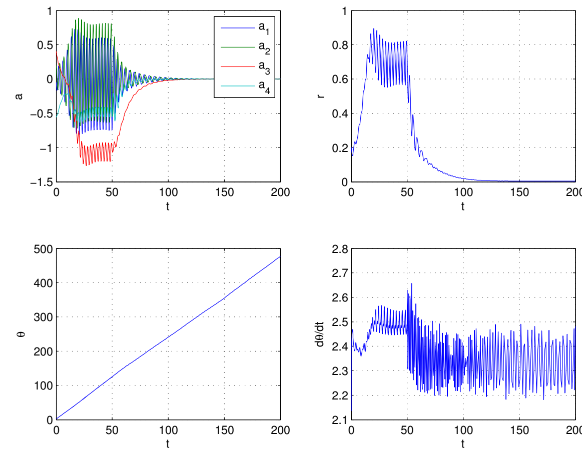

Fig. 6. Controlled response of the high order Galerkin model.

in the (ϕ, A) plane corresponding to the frequency conditions (Theorem 3) and the curves corresponding to the magnitude condition (Theorem 4) are plotted in Figure 4. The red zone is where the frequency criterion is satisfied, and the green zone is where the magnitude criterion is satisfied. The control parameters we seek are those that will result in a response satisfying both criteria, which corresponds to the intersection of the zones, which appears as brown in the figure.[5] The point $(\phi, A) = (54°, 300)$, which is marked with a circle in the figure, lies in this region and hence $\gamma = 300r\cos(\theta - 54\pi/180)$ can be used as the control law. Figure 5 shows the closed loop response of the oscillatory modes, where the controller is turned on at $t = 50$ seconds. Figure 6 shows corresponding response for the high order Galerkin model. It can be seen that the controller designed on the simplified model, and then applied to the high order Galerkin system (which accurately represents the cavity flow) achieves the desired task and drives $r \to 0$, in addition to keeping $\dot{\theta}$ between $\omega_{\text{low}} = 2.14$ and $\omega_{\text{high}} = 2.89$. Note also the similarity in behavior between the simplified model and the higher order Galerkin system, which in a further encouragement. There are inevitably some differences, for instance, the response of the frequency $\dot{\theta}$ in the higher order Galerkin model shows some deviation from that of the simplified low order model. Still, the approximation is adequate and the control law succeeds in achieving the desired objectives.

[5]When printed in grayscale, green appears light, red appears dark and brown appears slightly darker.

V. Conclusions and Future Works

In this paper we studied the control of fluid flow problems where the control goal is to reduce the amplitude of undesired oscillations. This goal is to be realized under the constraint that the oscillation frequency must remain within a certain range at all times. A simplified model of the oscillatory modes was obtained from the flow dynamics, and rules for selecting the controller parameters achieving the desired objectives were derived. Cavity flow control was studied as a real-life example to illustrate the ideas developed. It was seen that the controller designed using the results developed in the paper was successful is suppressing cavity oscillations, while at the same lime keeping the frequency within desired limits.

The problem studied in the paper is of practical importance since many fluid flow configurations exhibit unwanted oscillations that need to be suppressed. However, most of the devices used for actuation and measurement can only operate within certain frequency limits, so they cannot physically realize arbitrary control laws. The results of the paper can provide guidelines into reducing unwanted oscillations through control laws that are actually feasible through such actuators and transducers.

Future work ideas and research directions include using frequency-based approximations to the PDEs, extending the ideas to different types of control laws and applying the results to different problems in flow control or possibly in other fields.

ACKNOWLEDGEMENTS

We thank Professors Andrea Serrani, Mo Samimy and all members of the OSU GDTL Flow Control Group for our collaborative works on cavity flow control. We also thank Professors Bernd Noack and Gilead Tadmor for fruitful and insightful discussions during the initial phases of our research. We are also grateful to the libraries of TOBB Economics and Technology University for providing valuable resources that has made this work possible.

REFERENCES

[1] M. Gad-el Hak. *Flow Control Passive, Active, and Reactive Flow Management*. Cambridge University Press, New York, NY, 2000.

[2] T.R. Bewley. Flow control: new challenges for a new Renaissance. *Progress in Aerospace Sciences*, 37(1):21–58, 2001.

[3] O. M. Aamo, M. Krstic, and T. R. Bewley. Control of mixing by boundary feedback in 2d channel flow. *Automatica*, 39(9):1597–606–, 2003. ISSN 0005-1098;.

[4] K. Cohen, S. Siegel, T. McLaughlin, E. Gillies, and J. Myatt. Closed-loop approaches to control of a wake flow modeled by the Ginzburg-Landau equation. *Computers & Fluids*, 34(8):927–49–, 2005. ISSN 0045-7930;.

[5] B. R. Noack, K. Afanasiev, M. Morzynski, G. Tadmor, and F. Thiele. A hierarchy of low-dimensional models for the transient and post-transient cylinder wake. *Journal of Fluid Mechanics*, 497:335–63, 2003. ISSN 0022-1120;.

[6] B. R. Noack, P. Papas, and P. A. Monketwitz. The need for a pressure-term representation in empirical Galerkin models of incompressible shear flows. *Journal of Fluid Mechanics*, 523:339–65, 2005.

[7] K. Fitzpatrick, Y. Feng, R. Lind, A. J. Kurdila, and D. W. Mikolaitis. Flow control in a driven cavity incorporating excitation phase differential. *Journal of Guidance, Control, and Dynamics*, 28(1):63–70, 2005.

[8] M. Samimy, M. Debiasi, E. Caraballo, A. Serrani, X. Yuan, J. Little, and J. H. Myatt. Feedback control of subsonic cavity flows using reduced-order models. *Journal of Fluid Mechanics*, 579:315–346, 2007.

[9] S. N. Singh, J. H. Myatt, G. A. Addington, S. Banda, and J. K. Hall. Optimal feedback control of vortex shedding using proper orthogonal decomposition models. *Transactions of the ASME. Journal of Fluids Engineering*, 123 (3):612–618, 2001.

[10] E. Caraballo, J. Little, M. Debiasi, and M. Samimy. Development and implementation of an experimental based reduced-order model for feedback control of subsonic cavity flows. *Journal of Fluids Engineering*, 129:813–824, 2007.

[11] J.L. Gilarranz, L.W. Traub, and O.K. Rediniotis. A new class of synthetic jet actuators: Part II: Application to flow separation control. *Journal of fluids engineering*, 127(2):377–387, 2005.

[12] C. W. Rowley, T. Colonius, and R. M. Murray. Model reduction for compressible flows using POD and Galerkin projection. *Physica D*, 189(1-2):115–29, 2004.

[13] P. Holmes, J.L. Lumley, and G. Berkooz. *Turbulence, Coherent Structures, Dynamical System, and Symmetry*. Cambridge University Press, Cambridge, 1996.

[14] L. Sirovich. Turbulence and the dynamics of coherent structures. *Quarterly of Applied Math.*, XLV(3):561–590, 1987.

[15] R. C. Camphouse. Boundary feedback control using Proper Orthogonal Decomposition models. *Journal of Guidance, Control, and Dynamics*, 28:931–938, 2005.

[16] M. O. Efe and H. Ozbay. Low dimensional modelling and Dirichlet boundary controller design for Burgers equation. *International Journal of Control*, 77(10):895–906, July 2004.

[17] C. Kasnakoglu, A. Serrani, and M. O. Efe. Control input separation by actuation mode expansion for flow control problems. *International Journal of Control*, 81(9):1475 – 1492, 2008.

[18] D.W. Jordan and P. Smith. *Nonlinear Ordinary Differential Equations: An Introduction to Dynamical Systems*. Oxford University Press, 1999.

[19] C. Kasnakoglu and A. Serrani. Attenuation of Oscillations in Galerkin Systems Using Center-Manifold Techniques. *European Journal of Control*, 13(5):529, 2007.

[20] C.W. Rowley and V. Juttijudata. Model-based control and estimation of cavity flow oscillations. In *IEEE conference on decision and control*, volume 44, page 512. IEEE, 2005.